

Exercise 1

Solve the differential equation.

$$y'' - y' - 6y = 0$$

Solution

This is a linear homogeneous ODE with constant coefficients, so its solutions are of the form $y = e^{rx}$.

$$y = e^{rx} \quad \rightarrow \quad y' = re^{rx} \quad \rightarrow \quad y'' = r^2e^{rx}$$

Plug these formulas into the ODE.

$$r^2e^{rx} - re^{rx} - 6(e^{rx}) = 0$$

Divide both sides by e^{rx} .

$$r^2 - r - 6 = 0$$

Solve for r .

$$(r - 3)(r + 2) = 0$$

$$r = \{-2, 3\}$$

Two solutions to the ODE are e^{-2x} and e^{3x} . By the principle of superposition, then,

$$y(x) = C_1e^{-2x} + C_2e^{3x},$$

where C_1 and C_2 are arbitrary constants.